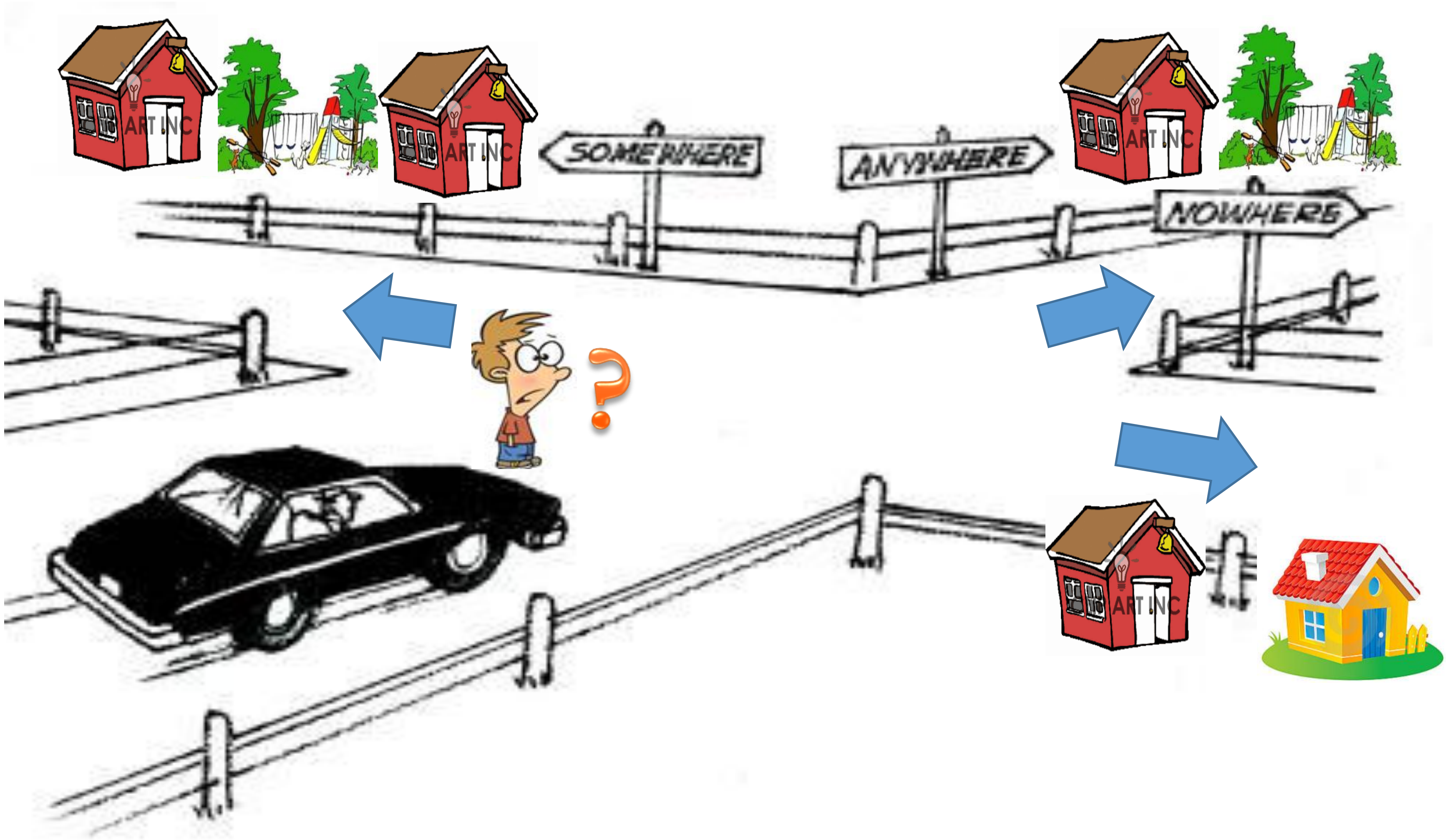
U.S. Political cartoon about finding a Republican presidential candidate in 1880

# Branch & Bound

Dr. Tamal Chakraborty

# Lost in a street corner



# Branch and Bound

1. Organize the solution space as a tree.
2. Systematically explore all the branches one by one (perform BFS).
3. Associate a cost with each branch and choose the branch with the lowest bound of the cost.
4. Discard (prune) those branches which won’t lead to the solution.

# The 15 Puzzle Problem

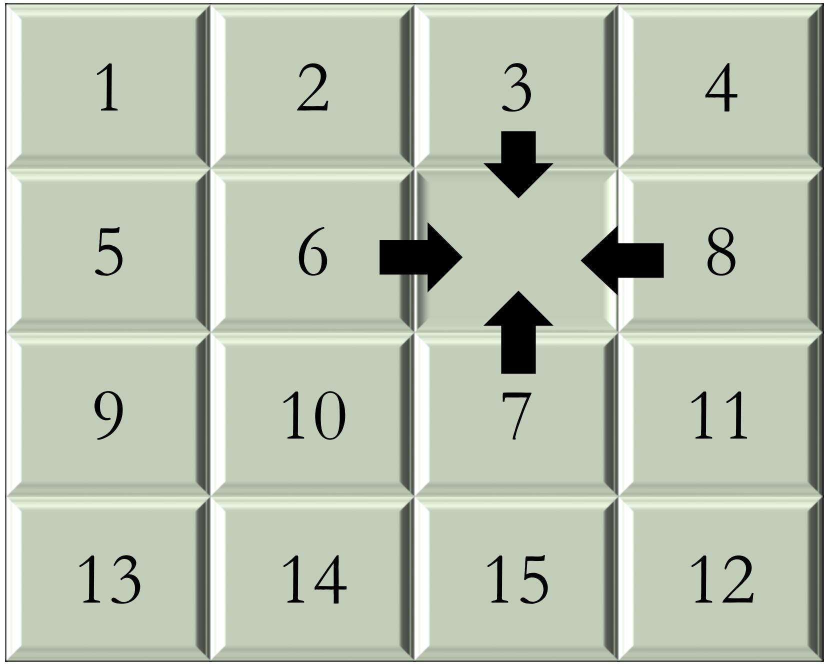
| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 | | 8 |
| 9 | 10 | 7 | 11 |
| 13 | 14 | 15 | 12 |

Initial State

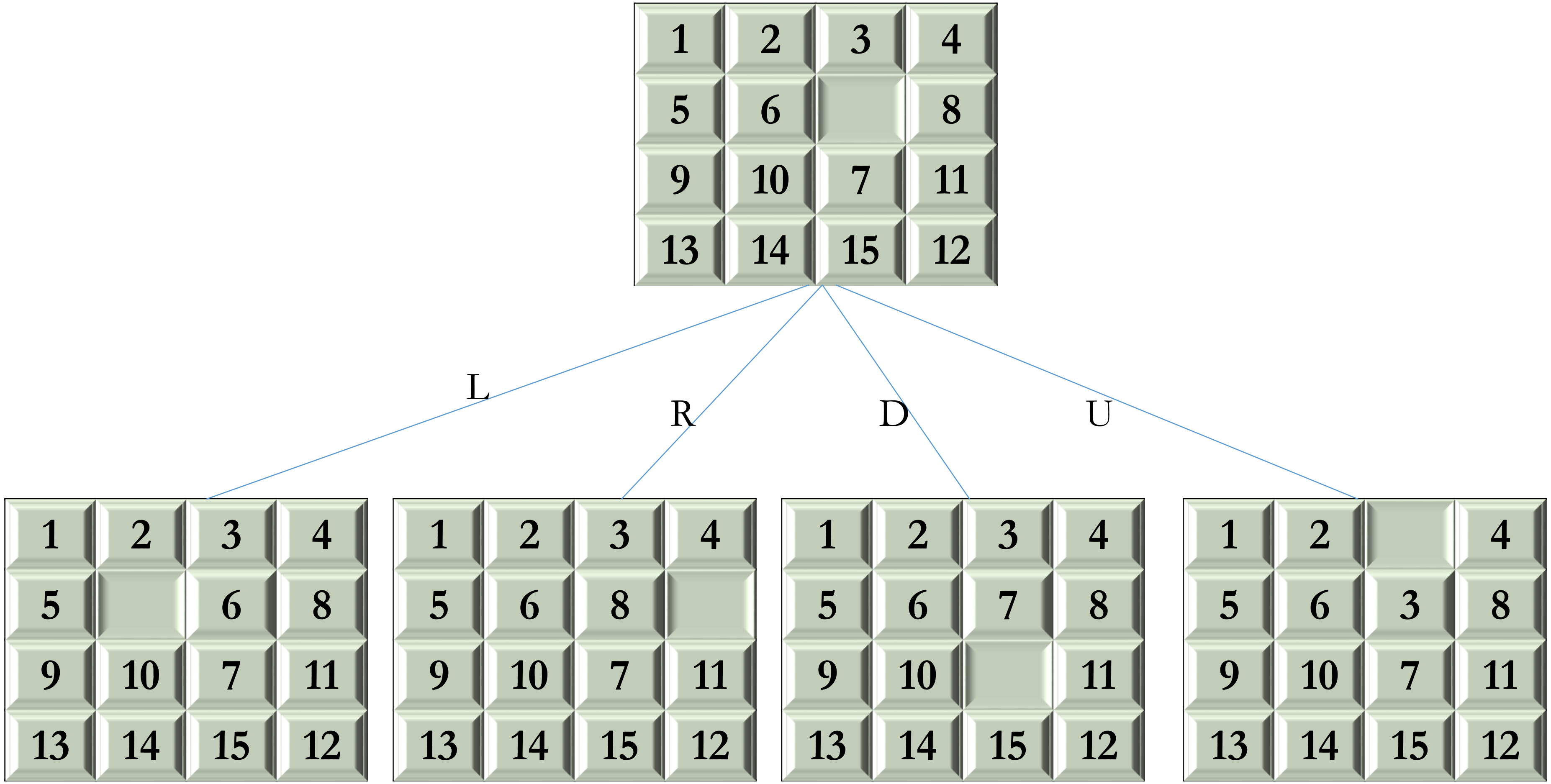
| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |
| 13 | 14 | 15 | |

Goal State

# Legal Moves



| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 | | 8 |
| 9 | 10 | 7 | 11 |
| 13 | 14 | 15 | 12 |



|  |  |  |  |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 |  | 8 |
| 9 | 10 | 7 | 11 |
| 13 | 14 | 15 | 12 |

|  |  |  |  |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 |  | 6 | 8 |
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|  |  |  |  |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
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| 9 | 10 | 7 | 11 |
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|  |  |  |  |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 |
| 9 | 10 |  | 11 |
| 13 | 14 | 15 | 12 |

|  |  |  |  |
|---|---|---|---|
| 1 | 2 |  | 4 |
| 5 | 6 | 3 | 8 |
| 9 | 10 | 7 | 11 |
| 13 | 14 | 15 | 12 |

Cost of a state = Distance from initial state + Distance from goal state

# Distance From GOAL State = Number of tiles NOT in GOAL position

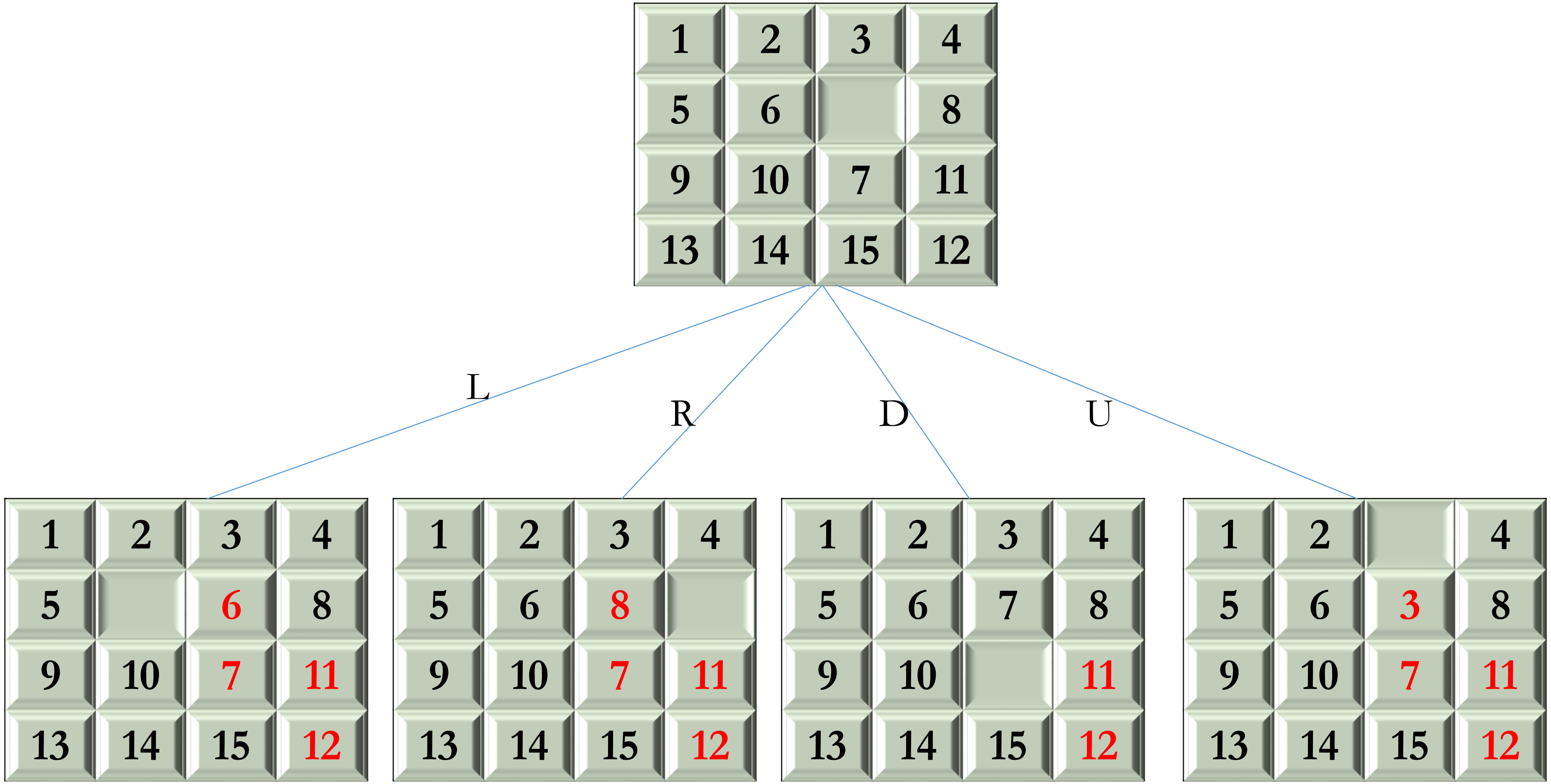
|   |   |   |   |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 |   | 8 |
| 9 | 10 | 7 | 11 |
| 13 | 14 | 15 | 12 |

Initial State

|   |   |   |   |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 |
| 13 | 14 | 15 |   |

Goal State

distance from
goal = 3



|  |  |  |  |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 |  | 8 |
| 9 | 10 | 7 | 11 |
| 13 | 14 | 15 | 12 |

L

|  |  |  |  |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 |  | 6 | 8 |
| 9 | 10 | 7 | 11 |
| 13 | 14 | 15 | 12 |

Cost = 1 + 4 = 5

R

|  |  |  |  |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 | 8 |  |
| 9 | 10 | 7 | 11 |
| 13 | 14 | 15 | 12 |

Cost = 1 + 4 = 5

D

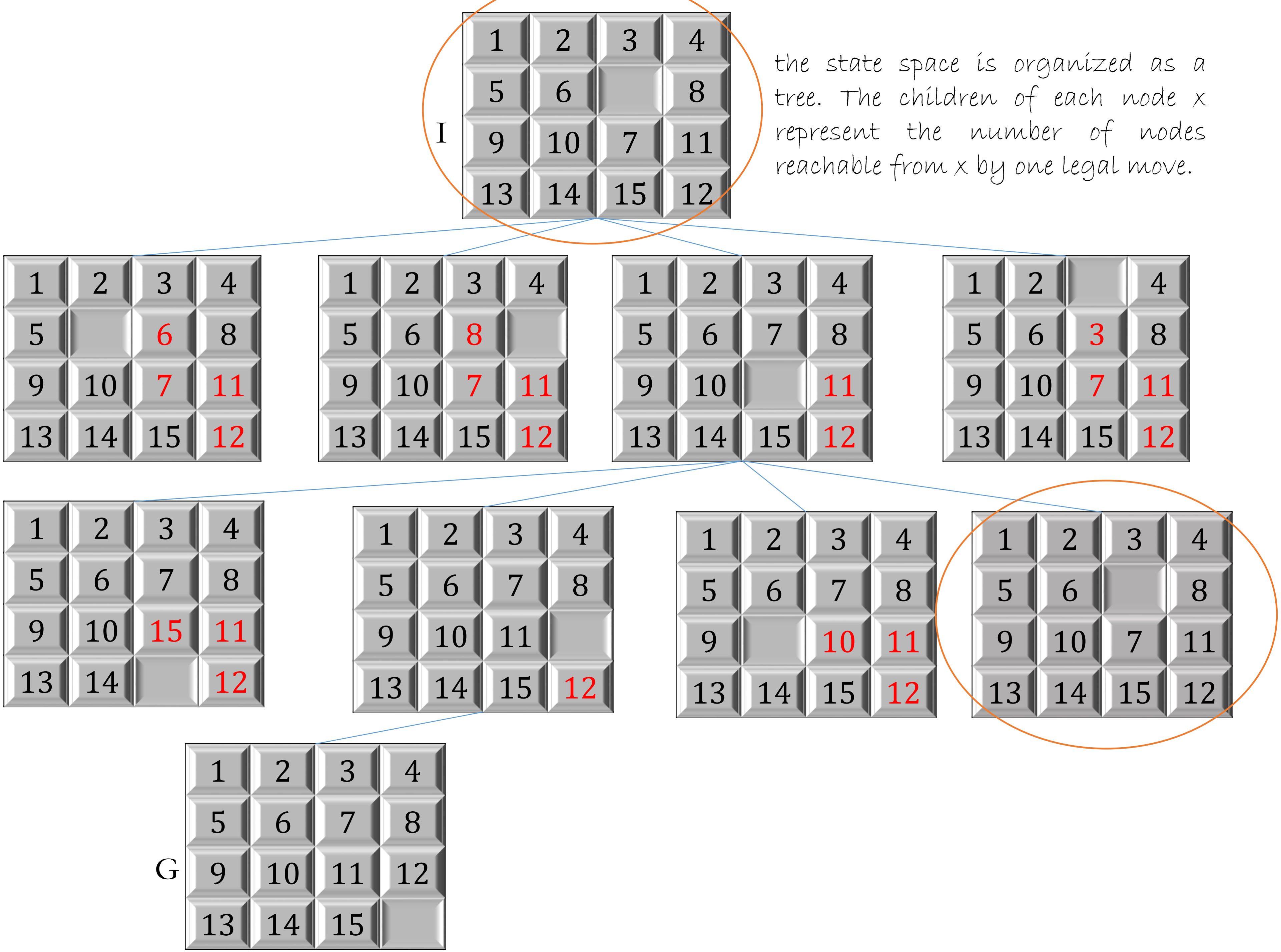
|  |  |  |  |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | 6 | 7 | 8 |
| 9 | 10 |  | 11 |
| 13 | 14 | 15 | 12 |

Cost = 1 + 2 = 3

U

|  |  |  |  |
|---|---|---|---|
| 1 | 2 |  | 4 |
| 5 | 6 | 3 | 8 |
| 9 | 10 | 7 | 11 |
| 13 | 14 | 15 | 12 |

Cost = 1 + 4 = 5



the state space is organized as a tree. The children of each node x represent the number of nodes reachable from x by one legal move.

# The Branch & Bound Algorithm for 15-puzzle

```
bnb15Puzzle(currentState)
  while (currentState ≠ GOAL)
      cost = ∞
      for each (state S reachable from currentState by 1 legal move)
          if (S ≠ currentState.parent)
              S.distanceFromInitial = currentState.distanceFromInitial + 1
              costOfState = S.distanceFromInitial + S.tilesNotInGOALpos()
              if (costOfState < cost)
                  cost = costOfState
                  nextState = S
      currentState = nextState
```

# Can you solve this 15-puzzle problem?

in 1878 Sam Loyd, America's puzzle-expert, "drove the whole world crazy" with his newly "discovered" 14-15 puzzle (though some accounts state that, other authorities dispute this fact).

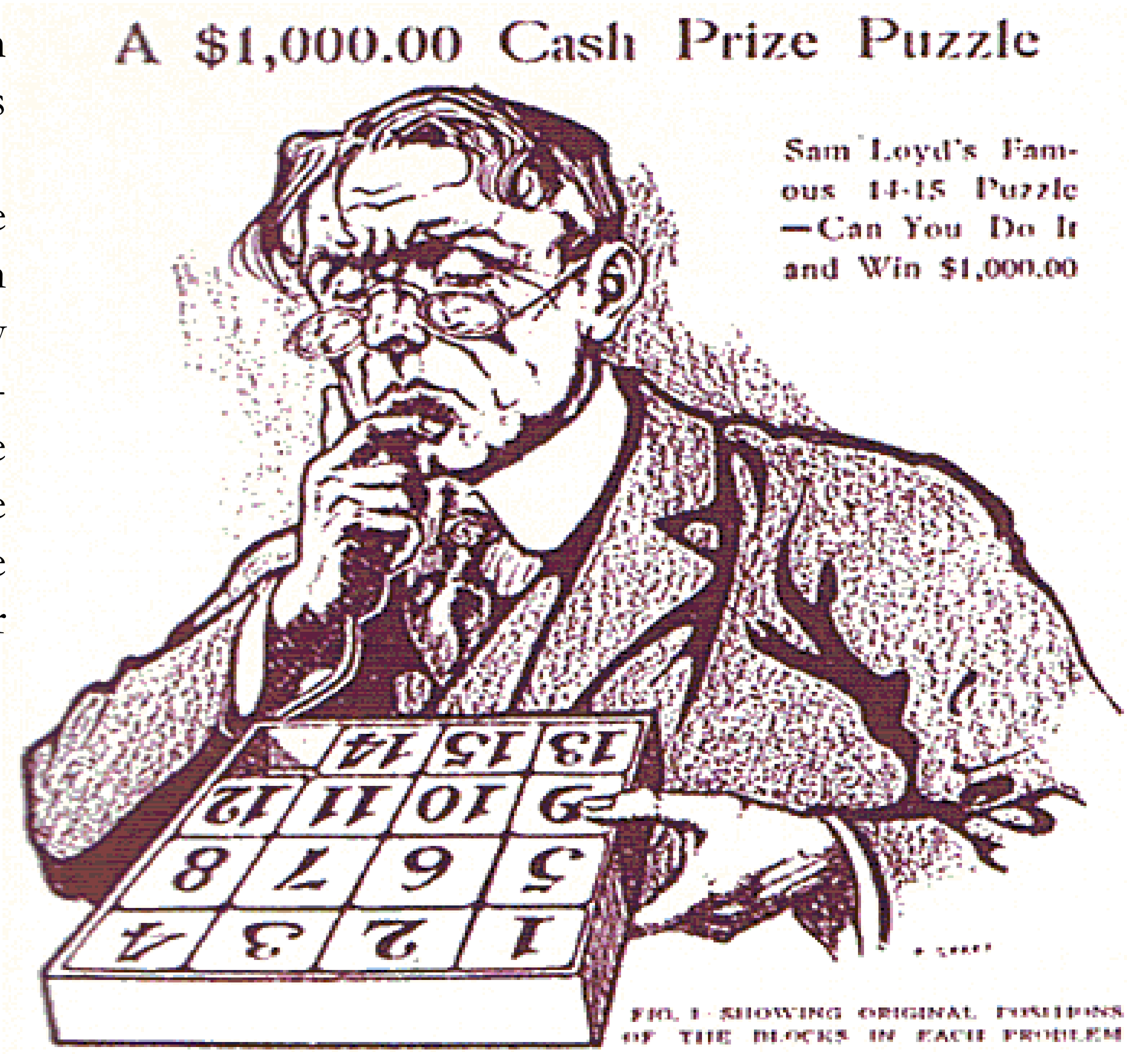

A $1,000.00 Cash Prize Puzzle
Sam Loyd's Famous 14-15 Puzzle —Can You Do It and Win $1,000.00
13 15 14
9 10 11 12
5 6 7 8
1 2 3 4

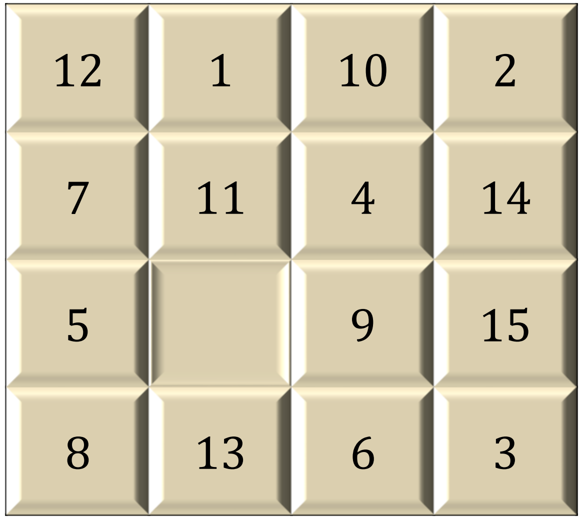
FIG. 1 - SHOWING ORIGINAL POSITIONS OF THE BLOCKS IN EACH PROBLEM

Sam Loyd claimed from 1891 until his death in 1911 that he invented the puzzle. Some later interest was fuelled by Loyd offering a $1,000 prize for anyone who could provide a solution for achieving a particular combination specified by Loyd, namely reversing the 14 and 15.

# How to check if a 15 Puzzle problem is solvable

## 1. Find the total number of inversions

An inversion is when a tile precedes another tile with a lower number on it. The goal state has zero inversions. For example, if, in a 4 x 4 grid, number 12 is top left, then there will be 11 inversions from this tile, as numbers 1-11 come after it. The table below gives the number of inversions for each tile in the puzzle.



| | | | |
|---|---|---|---|
| 12 | 1 | 10 | 2 |
| 7 | 11 | 4 | 14 |
| 5 | | 9 | 15 |
| 8 | 13 | 6 | 3 |

| **Tile** | **12** | **1** | **10** | **2** | **7** | **11** | **4** | **14** | **5** | **9** | **15** | **8** | **13** | **6** | **3** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nr. of inversions | 11 | 0 | 8 | 0 | 4 | 6 | 1 | 6 | 1 | 3 | 4 | 2 | 2 | 1 | 0 |

the total number of inversions in this puzzle is 49.

# How to check if a 15 Puzzle problem is solvable

## 2. Find the row number of the empty slot from bottom

We count the bottom-most row as row 1, and the top-most row as row 4. With this convention we note the row number of the empty slot is 2 in this game.

| | | | |
|---|---|---|---|
| 12 | 1 | 10 | 2 |
| 7 | 11 | 4 | 14 |
| 5 | | 9 | 15 |
| 8 | 13 | 6 | 3 |

A 15-puzzle problem is solvable if

The total number of inversions is odd and the empty slot is in an even row

or

The total number of inversions is even and the empty slot is in an odd row

Total number of inversions is 49 (odd). Empty slot is at row 2 (even).

This puzzle is solvable!!!

| | | | |
|---|---|---|---|
| 9 | 2 | 8 | 11 |
| | 5 | 13 | 7 |
| 15 | 1 | 4 | 10 |
| 3 | 14 | 6 | 12 |

*Thank You!*